



# **ST.ANNE'S COLLEGE OF ENGINEERING AND TECHNOLOGY**

(An ISO 9001:2015 Certified Institution)  
Anguchettypalayam, Panruti – 607106.

## **QUESTION BANK (R-2017)**

**MA3151  
MATRICES AND CALCULUS**



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**QUESTION BANK**

**PERIOD:** AUG - DEC 2021

**BATCH:** 2021 – 2025

**BRANCH:** COMMON

**YEAR/SEM:** I/01

**SUB CODE/NAME:** MA3151 - MATRICES AND CALCULUS

**UNIT-1 MATRICES**

**PART – A**

1. State Cayley- Hamilton theorem.
2. Find the sum and product of the Eigenvalues of the matrix  $A = \begin{pmatrix} 3 & 8 & 6 \\ 8 & 4 & 2 \\ 6 & 2 & 5 \end{pmatrix}$
3. Find the sum and product of the Eigenvalues of the matrix  $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$
4. The Eigen value of a matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  are 3 and 0, what is the third Eigen value?  
And find the product of the Eigen value?  
5. Find the sum and product of all the Eigenvalues of  $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ .
6. If 2 and 3 are the two eigenvalues of  $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$  then find the value of b.
7. The product of two Eigenvalues of the  $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$  is 16.find the third Eigenvalue.
8. Find the Eigenvalues of the matrix  $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ .
9. Find the Eigenvalues of  $3A+2I$ , where  $A= \begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix}$ .
10. If  $\lambda$  is an Eigen value of a matrix A, then  $\lambda^{-1}$  is the Eigen value of  $A^{-1}$ .
11. If  $\lambda$  is an Eigen value of a matrix A, then  $\lambda^2$  is the Eigen value of  $A^2$ .
12. Prove that the Eigen value of a orthogonal matrix are of unit modulus.
13. If the Eigen value of the matrix 3x3 are 2,3,1 then find the Eigen value of adjoint of A.
14. If 2,-1,-3 are the Eigen value of the matrix A,then find the Eigen value of  $A^2 - 2I$ .
15. If the sum of two Eigen values and trace of a 3x3 matrix A are equal, find the value of  $|A|$ .

**16.** Prove that  $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2zx = 0$  is indefinite.

**17.** Give the nature of a quadratic form whose matrix is  $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ .

**18.** What is the nature of the quadratic form  $x^2 + y^2 + z^2$  in four variables?

**19.** Discuss the nature of the quadratic form  $2x^2 + 3y^2 + 2z^2 + 2xy$ .

**20.** Write down the matrix corresponding to the quadratic form  $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$ .

## **PART-B**

### **CHAPTER-1.1** (8-MARKS)

1. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
2. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$
3. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$
4. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
5. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
6. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$
7. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$
8. Find the **Eigen values** and **Eigen vectors** for the matrix  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

**CHAPTER-1.2****(8-MARKS)**

1. Verify the **Cayley-Hamilton** theorem and also find  $A^{-1}$  for the matrix  $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$
2. Verify the **Cayley-Hamilton** theorem and also find  $A^{-1}$  for the matrix  $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \\ 2 & 1 & -2 \end{bmatrix}$
3. Verify the **Cayley-Hamilton** theorem and also find  $A^{-1}$  for the matrix  $\begin{bmatrix} -3 & 2 & 1 \\ 3 & -1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$
4. Verify the **Cayley-Hamilton** theorem and also find  $A^{-1}$  for the matrix  $\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$
5. Verify the **Cayley-Hamilton** theorem and also find  $A^{-1}$  for the matrix  $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$
6. Verify the **Cayley-Hamilton** theorem and also find  $A^4$  for the matrix  $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$
7. Use **Cayley-Hamilton theorem to find the value of**  

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \quad \text{Where } A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$
8. Verify the **Cayley-Hamilton** theorem and also find  $A^4$  for the matrix  $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

**CHAPTER-1.3****(16-MARKS)**

1. Reduce the quadratic form into the canonical by using orthogonal transform  
 $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$  and also find Rank, signature, Index
2. Reduce the quadratic form into the canonical by using orthogonal transform  
 $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$  and also find Rank, signature, Index
3. Reduce the quadratic form into the canonical by using orthogonal transform  
 $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$  and also discuss the nature

4. Reduce the quadratic form into the canonical by using orthogonal transform

$$3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_2x_3 + 2x_3x_1 \text{ and also discuss the nature}$$

5. Reduce the quadratic form into the canonical by using orthogonal transform

$$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2 \text{ and also find Rank, signature, Index}$$

6. Reduce the quadratic form into the canonical by using orthogonal transform

$$3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz. \text{ and also discuss the nature}$$

7. Reduce the quadratic form into the canonical by using orthogonal transform

$$x^2 + y^2 + z^2 - 2xy - 2yz - 2zx. \text{ and also find Rank, signature, Index}$$

8. Reduce the quadratic form into the canonical by using orthogonal transform

$$x_1^2 + 2x_2^2 + x_3^2 - 12x_1x_2 + 2x_2x_3 \text{ and also find Rank, signature, Index}$$

Reduce the quadratic form into the canonical by using orthogonal transform

$$2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz \text{ and also find Rank, signature, Index}$$



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**SUB CODE/NAME:** MA3151 - MATRICES AND CALCULUS

**UNIT- 2 DIFFERENTIAL CALCULUS**

**PART – A**

1. Find the domain and range  $f(x) = 3x - 2$ .
2. Sketch the graph of the absolute value function  $f(x) = |x|$
3. Prove that  $\lim_{x \rightarrow 0} |x| = 0$ .
4. If  $x^2 + y^2 = 25$ , then find  $\frac{dy}{dx}$ .
5. Find the derivative  $y = (x^3 - 1)^{100}$ .
6. Find the domain and range  $y = x^2$ .
7. Sketch the graph of function  $|x| = \begin{cases} x, & \text{if } x > 0 \\ -x, & \text{if } x < 0 \end{cases}$ .
8. Find the  $\lim_{x \rightarrow 3^+} \left( \frac{2x}{x-3} \right)$ .
9. Prove that  $\lim_{x \rightarrow 0} \left( \frac{|x|}{x} \right)$ .
10. Define derivative of a function  $f(x)$ .
11. Evaluate  $\lim_{x \rightarrow 1} \left( \frac{x^4 - 1}{x^3 - 1} \right)$ , if it exists.
12. Find the derivative of the function  $f(x) = \sqrt[3]{1 + \tan x}$ .
13. Sketch the graph of the function  $\begin{cases} 1 + x, & x < -1 \\ x^2, & -1 \leq x \leq 1 \\ 2 - x, & x \geq 1 \end{cases}$  and use it to determine the value of “a” for which  $\lim_{x \rightarrow a} f(x)$  exists? **(Jan-18)**
14. Does the curve  $y = x^4 - 2x^2 + 2$  have any horizontal tangents? If so where? **(Jan-18)**
15. State Rolle’s Theorem and verify the Rolle’s theorem for  $f(x) = x^3 + 5x^2 - 6x$  on the interval  $(0, 1)$ .
16. State Mean value theorem
17. Find the critical numbers for  $f'(x) = \frac{x^2(x-1)}{x+2}$ ,  $x \neq -2$
18. Define concavity and point of inflection.

19. Define maxima and minima of one variable and write the conditions.

20. Find the tangent line and normal line to the given curve  $y = 2xe^x$  at  $(0, 0)$ .

21. Find the domain of  $f(x) = \sqrt{3-x} - \sqrt{2+x}$  (Nov 2018)

22. Evaluate  $\lim_{t \rightarrow 1} \frac{t^4-1}{t^3-1}$  (Nov 2018)

23. check whether  $\lim_{t \rightarrow 1} \frac{3x+9}{|x+3|}$  exist. (APR 19)

24. Find the critical points of  $y = 5x^3 - 6x$  (APR 19)

## PART – B

### Limits

1. Find the value of  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ .

2. Find the domain of the functions a)  $y = x^2$ , b)  $f(x) = \sqrt{x-2}$ , c)  $g(x) = \frac{1}{x^2-x}$

3. Find  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} \right)$ .

4. Evaluate  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$ .

5. Find the limit of the function  $\lim_{x \rightarrow 0} \frac{e^{5x}-1}{x}$ , given numbers  $x = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.001$  (correct 6 decimal places) (Nov 2018)

### Continuity

1. Show that the function  $f(x) = 1 - \sqrt{1-x^2}$  is continuous on the interval  $[-1, 1]$ .

2. Find an equation of the tangent line to the parabola  $y = x^2$  at the point  $(1, 1)$ .

3. Find an equation of the tangent line to the hyperbola  $y = \frac{3}{x}$  at the point  $(3, 1)$ .

4. If  $(x) = \sqrt{x}$ , find the equation for  $f'(x)$ .

5. Determine whether  $f'(0)$  exist or not for the given function

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

6. Where is the function  $f(x) = |x|$  is differentiable?

7. Find  $\lim_{t \rightarrow 0} \frac{\sqrt{t^2+9}-3}{t^2}$ .

8. Find the value of  $\lim_{x \rightarrow 1} \left( \frac{x-1}{x^2-1} \right)$

9. For what value of the constant "c" is the function "f" continuous on  $(-\infty, \infty)$ ,

$$f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$

10. For what value of the constant "b" is the function "f" continuous on  $(-\infty, \infty)$ ,

$$\text{if } f(x) = \begin{cases} bx^2 - 2x & \text{if } x < 2 \\ x^3 - bx & \text{if } x \geq 2 \end{cases} \quad (\text{Apr 19})$$

### Differentiability

1. If the function  $f(x)$  is differentiable at  $a$ , then  $f(x)$  is continuous at  $a$ .

2. Find an equation of the tangent line to the hyperbola  $y = \frac{3}{x}$  at the point  $(3,1)$ .

3. Find an equation of the tangent line to the parabola  $y = x^2 - 8x + 9$  at the point  $(3,-6)$ .

4. Where is the function  $f(x) = |x|$  differentiable?

5. Show that  $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$ .

6. Show that the sum of  $x$  and  $x$  intercepts of any tangent line to the curve  $\sqrt{x} + \sqrt{y} = \sqrt{c}$  is equal to  $c$ .

7. Verify that the function  $f(x) = 5 - 12x + 3x^2$  satisfies the Rolle's theorem on the interval  $[1,3]$ .

8. Find the local maximum and local minimum values of the function  $g(x) = x + 2\sin x$ .

9. Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1, \quad -\frac{1}{2} \leq x \leq 4.$$

10. Find the local maximum and local minimum values of the function

$$f(x) = 2x^3 + 3x^2 - 36x.$$

11. Find the values of  $a$  and  $b$  such that the function  $f(x) = \begin{cases} x+2, & x < 2 \\ ax^2 - bx + 3, & 2 \leq x < 3 \\ 2x - a + b, & x \geq 3 \end{cases}$

is continuous everywhere.

12. Find the derivative of the function  $f(x) = \frac{1}{\sqrt{x}}$  using the definition of derivative. 0

13. Find the values of  $a$  and  $b$  such that the function  $f(x) = \begin{cases} \frac{x^3 - 8}{x - 2}, & x < 2 \\ ax^2 - bx + 3, & 2 \leq x < 3 \\ 2x - a + b, & x \geq 3 \end{cases}$  (Nov 2018)

14. find the derivative of  $f(x) = \cos^{-1} \left[ \frac{b+a \cos x}{a+b \cos x} \right]$  (Nov 2018)

15. find  $y'$  for  $\cos(xy) = 1 + \sin y$  (Nov 2018)

16. Find  $\frac{dy}{dx}$  if  $y = x^2 e^{2x} (x^2 + 1)^4$  (Apr 19)

### Maxima and minima

1. Find the maximum and minimum values of  $f(x) = \frac{x^2 + x + 1}{x^2 - x + 1}$ .

2. Find the local maximum and minimum values of  $f(x) = \sqrt{x} - 4\sqrt[4]{x}$  using both the first and second derivative tests. **(Jan-18)**
3. Find  $y''$  if  $x^4 + y^4 = 6$ . **(Jan-18)**
4. Find the tangent line to the equation  $x^3 + y^3 = 6xy$  at the point (3, 3) and at what point the tangent line horizontal in the first quadrant. **(Jan-18)**
5. For the function  $f(x) = 2 + 2x^2 - x^4$  find the intervals of increase or decrease , local maximum and minimum values, concavity and inflection points **(Nov 2018)**
6. For the function  $f(x) = 2x^3 + 3x^2 - 36x$  find the intervals of increase or decrease, local maximum and minimum values. **(Apr 19)**



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**UNIT- 3 FUNCTIONS OF SEVERAL VARIABLE**

**PART – A**

1. If  $\mathbf{u} = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$  then find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = ?$
2. If  $x^y + y^x = 1$ , then find  $\frac{dy}{dx} = ?$
3. If  $\mathbf{u} = x^2 + y^2$  and  $x = at^2$ ,  $y = 2at$ , find  $\frac{du}{dt}$ .
4. If  $x = r\cos\theta$ ,  $y = r\sin\theta$ , then find  $\frac{\partial(x,y)}{\partial(r,\theta)}$ .
5. If  $\mathbf{u} = \frac{y^2}{x}$ ,  $\mathbf{v} = \frac{x^2}{y}$ , find  $\frac{\partial(u,v)}{\partial(x,y)}$ .
6. If  $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \mathbf{0}$
7. If  $\mathbf{u} = \frac{y}{z} + \frac{z}{x}$  then find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = ?$
8. If  $\mathbf{u} = \mathbf{x}+\mathbf{y}$  and  $\mathbf{y} = \mathbf{u}\mathbf{v}$ , find the Jacobian  $\frac{\partial(x, y)}{\partial(u, v)}$ .
9. Write down Taylor's formula.
10. Find  $dy/dx$  when  $x^2+y^2 = xy$ .
11. Are  $u = x + y$  and  $v = x - y$  functionally independent? Justify the claim.
12. If  $x = r\cos\theta$ ,  $y = r\sin\theta$ , then find  $\frac{\partial r}{\partial x}$  (Jan-18)
13. If  $x = uv$ ,  $y = \frac{u}{v}$ , find  $\frac{\partial(x,y)}{\partial(u,v)}$ . (Jan-18)
14. Find  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$  if  $u = y^x$
15. If  $u = (x-y)(y-z)(z-x)$ , prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .
16. If  $u = (x-y)^4 + (y-z)^4 + (z-x)^4$  show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
17. Find  $\frac{du}{dt}$  if  $u = \frac{x}{y}$  where  $x = e^t$ ,  $y = \log t$
18. Find  $\frac{du}{dx}$  if  $u = \sin(x^2 + y^2)$ , where  $3x^2 + y^3 = 4$

19. If  $\mathbf{x} = \mathbf{u}^2 - \mathbf{v}^2$ ,  $\mathbf{y} = 2\mathbf{uv}$  evaluate the Jacobian of  $\mathbf{x}, \mathbf{y}$  with respect to  $\mathbf{u}, \mathbf{v}$  (Apr19)

20. If  $x^2 + y^2 = 1$ , then find  $\frac{dy}{dx}$ .

21. Find  $\frac{dy}{dx}$  if  $x^y + y^x = c$ , where  $c$  is a constant (Nov18)

22. State the properties of jacobians (Nov18)

23. Find  $\frac{du}{dt}$  in terms of  $t$ , if  $x^3 + y^3 = u$  where  $x = at^2, y = 2at$  (Apr19)

## PART – B

### Implicit functions

1. If  $u = \sin^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$  then prove that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$ .

2. If  $g = \Psi(u, v)$  where  $u = x^2 - y^2$  and  $v = 2xy$ ,

show that  $\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2 + y^2) \left[ \frac{\partial^2 \Psi}{\partial u^2} + \frac{\partial^2 \Psi}{\partial v^2} \right]$

3. If  $\mathbf{u} = f(x - y, y - z, z - x)$  then show that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ .

4. If  $\emptyset = \emptyset(u, v)$  where  $u = e^x \cos y, v = e^x \sin y$

Show that  $\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = (u^2 + v^2) \left[ \frac{\partial^2 \emptyset}{\partial u^2} + \frac{\partial^2 \emptyset}{\partial v^2} \right]$ .

5. If  $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$  then prove that  $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ .

6. If  $\mathbf{u} = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$ , then find the value of  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ ,

using Euler's theorem.

7. If  $z = f(x, y)$ , where  $x = e^u \cos v$  and  $y = e^u \sin v$  then show that  $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u} \frac{\partial z}{\partial y}$ .

8. If  $u = (x^2 + y^2 + z^2)^{-1/2}$  then find the value of  $u_{xx} + u_{yy} + u_{zz}$ . (Jan-18)

9. If  $\mathbf{u} = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$  find  $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$ . (Nov-18)

10. If  $\mathbf{u} = f(2x - 3y, 3y - 4z, 4z - 2x)$  then find  $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z}$  (Apr19)

### Maxima and minima

11. Find the Maximum and Minimum of  $f(x, y) = x^2 - xy + y^2 - 2x + y$ .

12. Find the Maximum and Minimum of  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ .

13. Find the Maximum and Minimum of  $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ . (Nov-18)

14. Examine  $f(x, y) = x^3 - 15y^2 - 15x^2 + 3xy^2 + 72x$  for extreme values. (Apr19)

### Jacobian

15. If  $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$  then find  $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$ .

16. If  $x + y + z = u, y + z = uv, z = uvw$ , prove that  $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$ .

17. Find the maximum and minimum values of  $f(x, y) = 3x^2 - y^2 + x^3$ . (Jan-18)

### Lagrange's multiplier method

18. A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box requiring, the least material for the construction.
19. The temperature at any point  $(x, y, z)$  in space is  $T = 400xyz^2$ . Find the highest temperature on the surface of the unit sphere  $x^2 + y^2 + z^2 = 1$ .
20. Find the shortest and the longest distances from the point  $(1, 2, -1)$  to the sphere  $x^2 + y^2 + z^2 = 24$ . (Apr19)
21. Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm. (Jan-18)
22. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ using Lagrange's method.}$$

23. Find the shortest distances from the point  $(1, 2, 0)$  to the cone  $x^2 + y^2 = z^2$ . (Nov-18)

### Taylor's series method

24. Expand  $f(x, y) = e^x \cos y$  at  $\left(0, \frac{\pi}{2}\right)$  upto 3<sup>rd</sup> term using Taylor's series.
25. Obtain the Taylor's series expansion of  $x^3 + y^3 + xy^2$  in terms of power of  $(x-1)$  and  $(y-2)$  upto third degree terms. (Jan-18)
26. Find the taylor's series of function  $f(x) = \sqrt{1+x+y^2}$  in powers  $(x-1)$  and  $y$  upto second degree terms. (Nov-18)
27. Expand Taylor's series expansion of  $x^2y^2 + 2x^2y + 3xy^2$  in terms of power of  $(x+2)$  and  $(y-1)$  upto third degree terms. (Apr19)



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**QUESTION BANK**

**PERIOD:** AUG - DEC 2021

**BATCH:** 2021 – 2025

**BRANCH:** COMMON

**YEAR/SEM:** I/01

**SUB CODE/NAME:** MA3151 - MATRICES AND CALCULUS

**UNIT-4 INTEGRAL CALCULUS**

**PART – A**

1. Find the derivative of  $\mathbf{g}(x) = \int_0^x \sqrt{1+t^2} dt$ .
2. Find the derivative of  $\mathbf{g}(x) = \int_a^x (t^3 + 1) dt$ .
3. Evaluate  $\int_{-1}^2 (x^3 - 2x) dx$  using Fundamental theorem of calculus.
4. Evaluate  $\int_1^4 (5 - 2t + 3t^2) dt$  using Fundamental theorem of calculus.
5. Evaluate  $\int e^{x^3} x^2 dx$ .
6. Evaluate  $\int e^{\cos x} \sin x dx$ .
7. Evaluate  $\int x \sin x dx$ .
8. Evaluate  $\int t e^t dt$ .
9. Write down reduction formula of  $\int_0^{\frac{\pi}{2}} \sin^n x dx$ .
10. Evaluate the improper integral  $\int_0^{\infty} \frac{1}{x} dx$ .
11. Evaluate  $\int \cos^2 x dx$ .
12. Evaluate  $\int \frac{1}{\sqrt{a^2-x^2}} dx$ .
13. What is wrong with the equation  $\int_{-1}^2 \frac{4}{x^3} dx = \left[ \frac{-2}{x^2} \right]_{-1}^2 = \frac{3}{2}$ ? **(Jan-18)**
14. Evaluate  $\int_4^{\infty} \frac{1}{\sqrt{x}} dx$  and determine whether it is convergent or divergent. **(Jan-18)**
15. Use the properties of integrals to evaluate  $\int_0^4 (4 + 3x^2) dx$ .
16. Write the substitution rule and solve  $\int \frac{x}{\sqrt{1-4x^2}} dx$ .

17. Determine whether  $\int_0^{\frac{\pi}{2}} \sec x dx$  converges or diverges.

18. Define improper integral for discontinuous integrands.

19. Determine whether the integral  $\int_1^{\infty} \frac{1}{x^2} dx$  is convergent or divergent.

20. Evaluate  $\int \frac{1}{x\sqrt{x+9}} dx$ .

21. State the fundamental theorem of calculus. (Nov18)

22. If  $f$  is continuous and  $\int_0^4 f(x) dx = 10$ , find  $\int_0^2 f(2x) dx$  (Nov18)

23. Evaluate  $\int \frac{1}{1+\tan x} dx$ . limits 0 to  $\frac{\pi}{2}$  (Apr19)

24. Evaluate  $\int_3^{\infty} \frac{dx}{(x-2)^2}$  and determine whether it is convergent or divergent. (Apr19)

## PART – B

### **Definite integrals**

25. Evaluate  $\int_0^3 (x^2 - 2x) dx$  by using Riemann sum by taking right end points as the sample points.

26. Prove that  $\int_a^b x dx = \frac{b^2 - a^2}{2}$  by using Riemann sum by taking right end points as the sample points.

### **Indefinite integrals**

27. Evaluate  $\int \frac{\tan x}{\sec x + \cos x} dx$ . (Jan-18)

28. Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$  (Apr19)

29. Evaluate  $\int_0^{\pi/2} \cos^5 x dx$ . (Jan-18)

30. Evaluate the integrals 1)  $\int x^3 \cos(x^4 + 2) dx$  2)  $\int_1^2 \frac{1}{(3-5x)^2} dx$

31. Evaluate  $\int_0^{\frac{\pi}{4}} x \tan^2 x dx$  (Apr19)

### **Substitution methods**

32. Find  $\int \sin^n x dx$  using reduction formula and evaluate  $\int_0^{\frac{\pi}{2}} \sin^n x dx$ . (Nov18)

33. Evaluate  $\int e^{ax} \cos bx dx$  using integration by parts. (Jan-18)

34. Evaluate  $\int e^{-ax} \sin bx dx$  ( $a > 0$ ) using integration by parts. (Apr19)

35. Evaluate  $\int (\log x)^2 dx$  using integration by parts

36. Evaluate  $\int e^x \sin x dx$ .

37. Evaluate  $\int \tan^{-1} x dx$ . Also find  $\int_0^1 \tan^{-1} x dx$

38. Evaluate  $\int e^{\tan^{-1} x} \left( \frac{1+x+x^2}{1+x^2} \right) dx$ .

39. Evaluate  $\int \frac{(\log x)^2}{x^2} dx$  using integration by parts. (Nov18)

40. Evaluate  $\int \tan^{-1} x dx$ . Also find  $\int_0^1 \tan^{-1} x dx$ .

### Improper integrals

41. For what values of 'p' is the integral  $\int_1^\infty \frac{1}{x^p} dx$  convergent? (Nov18)

42. Determine whether the integral  $\int_1^\infty \frac{\log x}{x^2} dx$  is convergent or divergent.

### Partial fraction method

43. Evaluate  $\int \frac{10}{(x-1)(x^2+9)} dx$ .

44. Evaluate  $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} dx$ .

45. Evaluate  $\int \frac{x^2+x+1}{(x-1)^2(x-2)} dx$ , by using partial fraction.

### Integral of the form

46. Evaluate  $\int (3x - 2) \sqrt{x^2 + x + 1} dx$

47. Evaluate  $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{1}{x^5 \sqrt{9x^2-1}} dx$  (Nov18)

48. Evaluate  $\int \frac{x}{\sqrt{x^2 + x + 1}} dx$ . (Jan-18)

49. Evaluate  $\int \frac{2x+5}{\sqrt{x^2-2x+10}} dx$  (Apr19)



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**UNIT -5 MULTIPLE INTEGRAL**

**PART – A**

1. Sketch the region of integration of the of the integral  $\int_0^1 \int_0^x f(x, y) dy dx$  and change the order of integration.
2. Sketch the region of integration of the of the integral  $\int_0^\infty \int_0^y ye^{-y^2/x} dx dy$  and change the order of integration.
3. Evaluate integral  $\int_0^1 \int_1^2 x(x + y) dy dx$ .
4. Evaluate integral  $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r d\theta dr$ .
5. Evaluate integral  $\int_0^{\pi} \int_0^a r d\theta dr$ .
6. Evaluate integral  $\int_1^2 \int_1^3 \frac{dxdy}{xy}$ .
7. Evaluate the triple integral  $\int_0^1 \int_0^2 \int_0^3 e^{x+y+z} dz dx dy$ .
8. Evaluate the triple integral  $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^1 r^2 \sin\theta dr d\theta d\varphi$ .
9. Change the order of integration in  $\int_0^1 \int_{x^2}^{2-x} f(x, y) dy dx$ .
10. Evaluate the triple integral  $\int_0^1 \int_0^y \int_0^{x+y} dz dx dy$ .
11. Find the value of  $\iint_{0,0}^{\infty, y} \left( \frac{e^{-y}}{y} \right) dx dy$  (Jan-18)
12. Find the limits of the integration in the double integral  $\iint_R f(x, y) dx dy$  where R is in the first quadrant and bounded by  $x=1$ ,  $y=0$ ,  $y^2=4x$ . (Jan-18)
13. Evaluate  $\iint_{0,0}^{3,2} e^{x+y} dy dx$
14. Find the area bounded by the lines  $x=0$ ,  $y=1$  and  $y=x$  using double integration.

15. Sketch the region of integration in  $\int_0^1 \int_0^x dx dy$

16. Transform into polar co-ordinates the integral  $\int_0^a \int_y^a f(x, y) dx dy$ .

17. Transform into polar co-ordinates the integral  $\int_0^\infty \int_0^\infty f(x, y) dy dx$

18. Write down the double integral to find area between the circles  $r = 2 \sin \theta$  and  $r = 4 \sin \theta$

19. Evaluate  $\int_0^a \int_0^b (x+y) dx dy$

20. Evaluate  $\int_0^{4x^2} \int_0^y e^{yx} dy dx$

$$\int_1^{\log 8} \int_0^{\log y} e^{x+y} dx dy \quad (\text{Nov18})$$

21. Evaluate

22. Change the order of integration in  $\int_0^1 \int_{y^2}^y f(x, y) dx dy \quad (\text{Nov18})$

23. Evaluate  $\int_1^a \int_2^b \frac{1}{xy} dx dy \quad (\text{Apr19})$

24. Find the limits of integraton  $\iint f(x, y) dx dy$  bounded by  $x = 0, y = 0, x + y = 2 \quad (\text{Apr19})$

## PART – B

### Double Integrals

1. Evaluate  $\iint xy dx dy$  over the region in the positive quadrant bounded by  $\frac{x}{a} + \frac{y}{b} = 1 \quad (\text{Apr19})$

2. Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  using polar coordinates

### Change of Order of Integration

3. Evaluate integral  $\int_0^1 \int_y^{2-y} xy dx dy$  by changing the order of integration.

4. Evaluate integral  $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy dx$  by changing the order of integration. (Jan-18)

5. Evaluate integral  $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$  by changing the order of integration (Apr19)

6. By changing the order of integration, evaluate  $\int_0^1 \int_y^1 \frac{x}{x^2 + y^2} dx dy$

7. Change the order of integration in the integral  $\int_0^1 \int_{x^2}^{2-x} xy dx dy$

### Area as Double Integral

8. Find the area which is inside the circle  $r = a \sin \theta$  but lying outside the cardioids  $r = a(1 - \cos \theta)$

9. Find by double integral, the area enclosed by the curves  $y^2 = 4ax$  and  $x^2 = 4ay$
10. Find the area of the Ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
11. Find the area which is inside the circle  $r = a \sin \theta$  but lying outside the cardioids  $r = a(1 - \cos \theta)$
12. Find by double integral, the area enclosed by the curves  $y=x$  and  $y=x^2$  **(Jan-18)**
13. Evaluate  $\iint xy(x+y)dx dy$  over area between  $y=x$  and  $y=x^2$  **(Nov18)**
14. Find the area bounded by the parabolas  $y^2 = 4 - x$  and  $y^2 = x$  **(Nov18)**

### Triple Integration

15. Evaluate  $\iiint x^2yz dx dy dz$  taken over the tetrahedron bounded by the planes  $x=0, y=0, z=0$  and  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
16. Evaluate  $\iiint_V \frac{1}{(x+y+z+1)^3} dx dy dz$  where V is the region bounded by  $x=0, y=0, z=0, x+y+z=1$ .
17. Evaluate  $\iiint xyz dx dy dz$  over the first octant of  $x^2+y^2+z^2=a^2$  **(Jan-18)**
18. Evaluate  $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$
19. Evaluate  $\iiint xyz dx dy dz$  through the positive spherical octant for which  $x^2 + y^2 + z^2 \leq a^2$  **(Apr19)**

### Change of Variables

20. Evaluate by changing into polar coordinates the integral  $\iint_0^a \frac{x}{x^2 + y^2} dx dy$  **(Jan-18)**
21. Transform the integral into polar coordinates and hence evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy dx$
22. Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing to polar co-ordinates and hence  
find the value of  $\int_0^\infty e^{-x^2} dx$ .
23. Express  $\iint_0^a \frac{x^2}{(x^2 + y^2)^{\frac{3}{2}}} dx dy$  in polar coordinates and then evaluate it **(Nov18)**
24. Evaluate by changing into polar coordinates the integral  $\iint_0^a \frac{x}{\sqrt{x^2 + y^2}} dx dy$  **(Apr19)**

### Volume

25. Using triple integration, find the volume of the sphere  $x^2 + y^2 + z^2 = a^2$
26. Find the volume of the tetrahedron bounded by the plane  $x + y + z = 1$  and the coordinate plane.
27. Evaluate  $\iiint dx dy dz$  where V is the finite region of space bounded by the planes  $x = 0, y = 0, z = 0$  and  $2x + 3y + 4z = 12$ . (tetrahedron) **(Nov18)**

